Default Logic Generalized and Simplified

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Dedicated to Victor Marek
on his 65th birthday

Abstract

We provide a description of generalized default logic as a unified formalism for nonmonotonic reasoning. Special attention will be paid to the role of the monotonic logic underlying default reasoning, as well as to the representation opportunities created by the use of assumptions (justifications) in the heads of default rules. On the other hand, it will be shown that even the generalized default logic can be simplified to a formal system that involves only monotonic rules and default assumptions.

1 Introduction

Default logic is now an almost thirty years old, so it is a good opportunity to review the present status of its development and try to understand its role in the general field of nonmonotonic reasoning.

Default logic has been born as just one of a number of alternative formalizations of nonmonotonic reasoning. In the course of its development, however, it has become increasingly clear that default logic occupies a special place in nonmonotonic reasoning, both with respect to its representation capabilities, and in its relations to other nonmonotonic formalisms.

The main objective of this study consists in confirming and elaborating the claim that default logic can serve as a general formalism for nonmonotonic reasoning, sufficient to deal with the majority of the tasks and problems posed to the latter by AI.

The plan of the paper is as follows. First, we describe a powerful generalization of the original, Reiter's default logic to disjunctive default rules that may contain, in addition, justifications in heads. Essentially, this generalization has been suggested first in (Lin & Shoham 1992; Lifschitz 1994) in a modal framework, and it subsumes disjunctive default logic of (Gelfond *et al.* 1991). Next, we will describe the formalism of default biconsequence relations that constitutes the logical basis of this generalized default logic. The latter will help us in showing, in particular, that both the autoepistemic logic (Moore 1985) and production and causal inference from (Bochman 2004) are subsumed by the latter. In both cases the differences between these formalisms and default logic are reducible to differences in the underlying logic. Finally, we will show that the generalized default logic is equivalent in expressive capabilities to the disjunctive default logic. Furthermore, both are reducible to a simplified formalism that contains only monotonic rules and default assumptions (aka supernormal defaults). This latter reduction will demonstrate also that default logic subsumes, in a sense, the general assumption-based formalism for nonmonotonic reasoning suggested in (Bondarenko *et al.* 1997).

2 Default Logic Generalized

Originally (see (Reiter 1980)) default theory was defined as a pair (W, D), where W is a set classical propositions (the axioms), and D a set of default rules of the form $A : b/C$, where A, C are propositions and b a finite set of propositions. Very informally, a rule $A : b/C$ was intended to state something like: 'if A is believed, and each $B \in b$ can be consistently believed, then C should be believed'.

Extensions of a default theory were defined by a fixed point construction: for a set u of propositions, let $\Gamma(u)$ be the least deductively closed set that includes W and satisfies the following condition:

- If $A : b/C$, $A \in \Gamma(u)$ and $\neg B \notin u$, for any $B \in b$, then $C \in \Gamma(u)$.

Then a set s is an *extension* of the default theory if and only if $\Gamma(s) = s$.

As can be seen, default claims are represented in default logic as inference rules affecting our beliefs. In this respect, Reiter's default logic has been largely inspired by the need to provide logical foundations for the procedural approach to nonmonotonicity found in deductive databases, logic programming and Doyle's truth maintenance. This representation avoids some of the problems arising with formula-based interpretations of defaults (such as contraposition of default claims). On the other hand, this representation makes default logic an inherently epistemic formalism. Namely, it primarily describes our beliefs and knowledge, unlike the extensional classical logic normally used for a direct representation of objective facts about the world. This epistemic understanding can be clearly discerned from the original description given in (Reiter 1980). For example, the very notation initially used by Reiter, namely $A{:}\mathbf{M}B_1, \ldots, \mathbf{M}B_n/C$, involved a rudimentary modal operator $\mathbf{M}$ employed for designating justifications. It was dropped later as syntactically unnecessary, though without changing the original understanding.

(Marek & Truszczyński 1989) have suggested a more logical description of default logic using the notion of a context-depended proof as a way of formalizing Reiter's operator Γ. This representation has been developed in (Marek, Nerode, & Remmel 1990) to a general theory of nonmonotonic rule systems (see also (Marek & Truszczyński 1993)).

Given a set s of propositions (the 'context'), let us consider the set $\mathcal{D}(s)$ of all propositions that are derivable from W using the classical entailment and the following ordinary inference rules:

$$\{A \vdash C \mid A : b/C \in D \ \& \ \neg B \notin s, \text{ for any } B \in b\}.$$

Then s is an extension of the default theory if and only if $s = \mathcal{D}(s)$.

The above representation makes it vivid that a large part of reasoning in default logic involves ordinary rule-based inference, the only distinction from traditional inference systems being that the very set of rules allowed in the inference process is determined by the (assumptions made in the) context.

An important generalization of default logic has been proposed in (Gelfond *et al.* 1991), guided by the need to provide a logical basis for disjunctive logic programming, as well as more perspicuous ways of handling disjunctive information. A disjunctive default theory is a set of *disjunctive defaults*, rules of the form $a : b/c$, where a, b, c are finite sets of formulas[1]. An informal meaning of such rules is 'If all propositions from a are believed, and each $B \in b$ can be consistently believed, then at least one proposition from c should be believed'.

Disjunctive default rules is actually a generalization of monotonic disjunctive rules $a \vdash b$, where a and b are sets of propositions. A set u of propositions is said to be *closed* with respect to a set R of disjunctive rules when, for any rule $a \vdash b$ from R, if $a \subseteq u$, then $b \cap u \neq \emptyset$. This generalized notion of deductive closure turns out to be a key to defining extensions of a disjunctive default theory.

For a set s of propositions, let $\mathcal{D}(s)$ denote the set of all minimal deductively closed theories that are closed also with respect to the following monotonic disjunctive rules:

$$\{a \vdash c \mid a : b/c \in D \ \& \ \neg B \notin s, \text{ for any } B \in b\}.$$

[1]This representation exploits the fact that the axioms W of a default theory are representable as rules of the form $\mathbf{t} : \emptyset/A$.

Then s is said to be an *extension* of a disjunctive default theory if $s \in \mathcal{D}(s)$.

A final generalization of interest for our present study has been suggested in (Lifschitz 1994; Lin & Shoham 1992) in a modal framework with two modal operators in an attempt to construct a unified formalism for nonmonotonic reasoning and logic programming. It has been shown in (Bochman 1995), however, that this formalism can be expressed in a purely non-modal setting by using rules of the form $a : b/c : d$, where a, b, c, d are sets of classical propositions. Such rules could be read as follows:

> *If all propositions from a are believed, and each proposition from b can be consistently believed, then at least one proposition from c should be believed, or else at least one proposition from d can be consistently believed'.*

Thus, compared with the preceding generalization to disjunctive default rules, the new default rules are disjunctive rules that may involve justifications also in their heads.

Below, for a set u of propositions, $\neg u$ will denote the set $\{\neg A \mid A \in u\}$. Then, in full analogy with the preceding constructions, the corresponding generalized default logic can be described as follows.

Definition 2.1. A generalized default theory is a set of rules $a : b/c : d$, where a, b, c, d are sets of classical propositions.

For a set s of propositions, let $\mathcal{D}(s)$ denote the set of minimal deductively closed theories that are closed also with respect to the rules

$$\{a \vdash c \mid a : b/c : d \in D \ \& \ \neg b \cap s = \emptyset \ \& \ \neg d \subseteq s.\}$$

Then s is an *extension* of a generalized default theory if $s \in \mathcal{D}(s)$.

An important feature of extensions of generalized default theories is that they already need not be minimal, so one extension may be a proper subset of another extension.

In effect, it has been shown already in (Lin & Shoham 1992) that an autoepistemic logic is representable in this formalism by using rules with justifications in heads. In addition, it has been suggested in (Lifschitz & Woo 1992) that generalized rules of this kind might be useful also in logic programming. And indeed, it has been shown in (Inoue & Sakama 1998) that program rules of the form

$$A, \mathbf{not}\, A \leftarrow$$

provide a faithful description of abducibles, so they can be used for a formal representation of abductive logic programming.

In what follows, we will show that both the autoepistemic logic and a more recent causal calculus (McCain & Turner 1997a; Turner 1999), are subsumed by the generalized default logic. In both cases, the generalization to default rules involving justifications in heads will turn out to be essential for an adequate representation. But first we will consider the underlying local basis of this default logic.

3 The Logic of Default Logic

A logical account of generalized default logic can be given in the framework of biconsequence relations. Biconsequence relations (see (Bochman 1998)) are specialized consequence relations for reasoning with respect to a pair of contexts. On the interpretation suitable for nonmonotonic reasoning, the first of these two contexts is the main (objective) one, while the other context provides assumptions that justify inferences in the main context. This separation of inferences and their justifications creates a framework for nonmonotonic reasoning.

A *bisequent* is an inference rule of the form $a : b \Vdash c : d$, where a, b, c, d are finite sets of propositions. An informal reading of such rules appropriate for our present purposes is as follows:

> If no proposition from b is assumed, and all propositions from d are assumed, then all propositions from a hold only if one of the propositions from c holds.

As can be seen from the above description, the meaning of bisequents is slightly different from the informal interpretation of generalized default rules $a{:}b/c{:}d$ given at the end of the preceding section. The reason for the difference is that bisequents allow for a more simple and transparent description of the associated logical system. Still, the correspondence between generalized default rules and bisequents is straightforward; namely, default rules $a : b/c : d$ are representable by bisequents of the form $a : \neg b \Vdash c : \neg d$.

A *biconsequence relation* is a set of bisequents satisfying the rules:

Monotonicity If $a \subseteq a'$, $b \subseteq b'$, $c \subseteq c'$, $d \subseteq d'$, then

$$\frac{a : b \Vdash c : d}{a' : b' \Vdash c' : d'}$$

Reflexivity $\quad A : \Vdash A :$ $\quad$ and $\quad$ $: A \Vdash : A$;

Cut $$\frac{a : b \Vdash A, c : d \quad A, a : b \Vdash c : d}{a : b \Vdash c : d}$$

$$\frac{a : b \Vdash c : A, d \quad a : A, b \Vdash c : d}{a : b \Vdash c : d}.$$

For a set u of propositions, $\overline{u}$ will denote the set of propositions that do not belong to u. A pair (u, v) of sets of propositions will be called a *bitheory* of a biconsequence relation if $u : \overline{v} \nVdash \overline{u} : v$. A set u of propositions is a *theory* of $\Vdash$, if (u, u) is a bitheory of $\Vdash$.

Bitheories can be seen as pairs of sets that are closed with respect to the bisequents of a biconsequence relation. A bitheory (u, v) of $\Vdash$ is *positively minimal*, if there is no bitheory (u', v) of $\Vdash$ such that $u' \subset u$. Such bitheories play an important role in describing nonmonotonic semantics.

By a *bimodel* we will mean a pair of sets of propositions. A set of bimodels will be called a *binary semantics.*

Definition 3.1. A bisequent $a : b \Vdash c : d$ is *valid* in a binary semantics $\mathcal{B}$, if, for any $(u, v) \in \mathcal{B}$, if $a \subseteq u$ and $b \subseteq \overline{v}$, then either $c \cap u \neq \emptyset$, or $d \cap \overline{v} \neq \emptyset$.

The set of bisequents that are valid in a binary semantics forms a biconsequence relation. On the other hand, any biconsequence relation $\Vdash$ is determined in this sense by its canonical semantics defined as the set of bitheories of $\Vdash$. Consequently, the binary semantics provides an adequate interpretation of biconsequence relations.

The nonmonotonic semantics of extensions for biconsequence relations is defined as follows:

Definition 3.2. A set u is an *extension* of a biconsequence relation $\Vdash$, if (u, u) is a positively minimal bitheory of $\Vdash$.

The above descriptions of biconsequence relations and the semantics of extensions are purely structural, so they are still not sufficient for capturing reasoning in default logic. To this end, we should 'upgrade' the formalism to a logical system that subsumes classical entailment.

An epistemic understanding of biconsequence relations is naturally obtained by treating the objective and assumption contexts, respectively, as the contexts of knowledge and belief. In other words, propositions that hold in the objective context can be viewed as known, while propositions belonging to the assumption context can be seen as forming the set of associated beliefs. Accordingly, both the objective and assumption contexts of bimodels will correspond in this case to deductively closed theories.

Supraclassical biconsequence relations, defined below, are biconsequence relations in a classical language such that both its component contexts respect classical entailment.

Definition 3.3. A biconsequence relation in a classical language will be called *supraclassical*, if it satisfies

Supraclassicality $$\frac{a \vDash A}{a : \Vdash A :} \qquad \frac{a \vDash A}{: A \Vdash : a}$$

Falsity $\quad \mathbf{f} : \Vdash$ $\quad$ and $\quad$ $\Vdash : \mathbf{f}$.

The most important consequence of Supraclassicality is the possibility of replacement of classically equivalent formulas in bisequents. In addition, it allows us to replace sets of objective premises and assumption sets in conclusions by their conjunctions, but objective conclusion sets and assumption sets in premises are not replaceable in this way by their classical disjunctions. Speaking more generally, supraclassical biconsequence relations are only *supra*-classical, which means, in particular, that the deduction theorem, contraposition, and disjunction in the antecedent are not valid, in general, for each of the two contexts. In addition, the conditions of Falsity impose a restriction to classically consistent theories[2].

A logical semantics of supraclassical biconsequence relations can be obtained from the general binary semantics by requiring that bimodels are pairs of consistent deductively closed sets. Such bimodels and semantics are called *classical*. Then we have (see (Bochman 2005))

Proposition 3.1. *A biconsequence relation is supraclassical if and only if it has a classical binary semantics.*

Recall that default rules $a : b/c : d$ are translated as bisequents $a{:}\neg b \Vdash c{:}\neg d$. Now, for a generalized default theory D, $\Vdash_D$ will denote the least supraclassical biconsequence relation that includes bisequents corresponding to the rules of D. Then the following result shows that biconsequence relations provide an adequate logical framework for default reasoning.

Theorem 3.2. *Extensions of a generalized default theory D coincide with extensions of $\Vdash_D$.*

The above representation theorem implies, in particular, that all the postulates of a supraclassical biconsequence relation are valid logical rules for default reasoning. The latter rules still do not constitute, however, a maximal logic underlying such a reasoning. Such a maximal logic can be defined as follows.

Definition 3.4. A *default biconsequence relation* is a supraclassical biconsequence relation that satisfies the following structural rules:

Consistency $\quad A : A \Vdash$

Regularity If $b : a \Vdash a : b$, then $: a \Vdash : b$.

Consistency correspond to the semantic requirement that $u \subseteq v$, for any bimodel (u, v), while regularity restricts the classical binary semantics to a *quasi-reflexive* semantics in which, for any bimodel (u, v), (v, v) is also a bimodel.

It turns out that the above defined default biconsequence relations constitute a maximal logic adequate for the generalized default logic. This fact can be demonstrated by showing that equivalence with respect to default biconsequence relations coincides with the strong equivalence for default theories.

Definition 3.5. Default theories D_1 and D_2 will be called *strongly equivalent*, if, for any set D of default rules, $D_1 \cup D$ has the same extensions as $D_2 \cup D$.

Originally, the notion of strong equivalence has been suggested in logic programming (see (Lifschitz, Pearce, & Valverde 2001)), but it turns out to have general significance. Strong equivalence is already a logical notion, since strongly equivalent theories are interchangeable in any larger theory without changing the associated semantics. And indeed the following result shows that strong equivalence amounts to logical equivalence equivalence with respect to default biconsequence relations.

Theorem 3.3. *Default theories are strongly equivalent if and only if they determine the same default biconsequence relation.*

In other words, default theories D_1 and D_2 are strongly equivalent if and only if each rule of D_2 is derivable from D_1 using the postulates of default biconsequence relation, and vice versa.

It is interesting to note that default biconsequence relations allow us to provide the following simplified description of extensions.

Proposition 3.4. *A set u is an extension of a default biconsequence relation $\Vdash$ if and only if*

$$u = \{A \mid : \overline{u} \Vdash A : u\}.$$

By the above description, an extension can be seen as a set of formulas that are provable on the basis of taking precisely itself as the set of assumptions. This description demonstrates, in particular, that generalized default logic is based essentially on the same ideas as the original Reiter's logic.

3.1 Saturation and autoepistemic logic

(Marek & Truszczyński 1989) introduced the notion of a *weak extension* of a default theory as a default counterpart of stable expansions in autoepistemic logic and models of Clark's completion in logic programming. Weak extensions of a default theory (W, D) can be defined as fixed points of a modified operator. For a set s of propositions, let $\Gamma_w(s)$ be the least deductively closed set that includes W and satisfies the following condition:

- If $A : b/C \in D$, $A \in s$ and $\neg B \notin s$, for any $B \in b$, then $C \in \Gamma_w(s)$.

Then a set s is a *weak extension* of the default theory if $\Gamma_w(s) = s$.

It can be shown indeed that (under a suitable translation), the above notion of a weak extension corresponds precisely to the notion of expansion in autoepistemic logic (Moore 1985). In this sense, the autoepistemic logic can be defined formally in a non-modal framework of Reiter's default rules by changing the associated nonmonotonic semantics. Unfortunately, this change in the nonmonotonic semantics is implicitly based on a more substantial change in the meaning of the default rules themselves. It should be clear that deliberations of this kind cannot even be stated precisely without considering the underlying logic of default reasoning. And indeed, it has been shown in (Bochman 1994) that both default and autoepistemic logic are definable in a single logical framework of default consequence relations by alternating the underlying logic of default rules. Moreover, it has been shown that the respective alternative underlying logics are incompatible on pain of trivialization.

[2] A fortiori, inconsistent extensions will be excluded from consideration.

In contrast to the above results (and in accordance with (Lin & Shoham 1992)), it can be shown that the autoepistemic logic is actually subsumed by the generalized default logic while preserving a single meaning of (generalized) default rules.

To begin with, it has been shown in (Bochman 2005) that the following nonmonotonic semantics for biconsequence relations constitutes an exact non-modal counterpart of Moore's autoepistemic logic.

As a preparation, note that any deductively closed theory u always contains maximal deductive sub-theories; such sub-theories are representable as sets $u \cap \alpha$, where α is a world (maximal deductive theory). Now, for a deductively closet set u, let $u\bot$ denote the set of all maximal sub-theories of u, plus u itself.

Definition 3.6. A theory u of a supraclassical biconsequence relation $\Vdash$ is a *classical expansion* of $\Vdash$, if, for any $v \in u\bot$ such that $v \neq u$, the pair (v, u) is not a bitheory of $\Vdash$. The set of classical expansions determines the *autoepistemic semantics* of $\Vdash$.

It follows directly from the respective definitions of extensions and expansions that any extension of a supraclassical biconsequence relation will be a classical expansion, though not vice versa. Still, it can be shown that expansions are precisely extensions of biconsequence relations under a stronger underlying logic described in the following definition.

Definition 3.7. A default biconsequence relation will be called *saturated*, if it satisfies the following postulate:

Saturation $\quad \Vdash A \lor B, \neg A \lor B : B$.

The the next definition provides a semantic description of Saturation.

Definition 3.8. A bimodel (u, v) will be called *saturated*, if $u \in v\bot$. A classical binary semantics $\mathcal{B}$ will be called *saturated* if its bimodels are saturated.

According to the above definition, a bimodel (u, v) is saturated, if u either coincides with v, or is a maximal theory included in v. The next result establishes completeness of saturated biconsequence relations with respect to the saturated binary semantics.

Proposition 3.5. *A biconsequence relation is saturated if and only if it has a saturated binary semantics.*

Now, it can be shown that the postulates of saturated biconsequence relations preserve expansions, so they are admissible for the autoepistemic semantics. Moreover, the next result shows that, for such biconsequence relations, expansions actually collapse to extensions.

Proposition 3.6. *Classical expansions of a saturated biconsequence relation coincide with its extensions.*

Actually, the next result shows that saturated biconsequence relations constitute a maximal logic for the autoepistemic semantics.

Theorem 3.7. *Two generalized default theories are strongly equivalent with respect to the autoepistemic semantics if and only if they determine the same saturated biconsequence relation.*

Our final result here states an important sufficient condition for coincidence of expansions and extensions of a default theory.

A generalized default theory D will be called *positively simple*, if objective premises and conclusions of any rule from D are sets of classical literals. Then we have[3]

Theorem 3.8. *Expansions of a positively simple default theory coincide with its extensions.*

Bisequents $a{:}b \Vdash c{:}d$ such that a, b, c, d are sets of classical literals, are logical counterparts of program rules of extended logic programs with classical negation (see (Gelfond & Lifschitz 1991; Lifschitz & Woo 1992)). The semantics of such programs is determined by *answer sets* that coincide with extensions of respective bisequent theories. Moreover, such bisequent theories are positively simple, so by Theorem 3.8 extended logic programs obliterate the distinction between extensions and expansions. This is the logical basis for a possibility of representing extended logic programs also in autoepistemic logic.

3.2 Causal biconsequence relations and causal logic

Production and causal inference relations have been introduced in (Bochman 2003) as a logical formalization of reasoning in causal theories of action and change (McCain & Turner 1997a). Such inference relations are based on conditionals of the form $A \Rightarrow B$ saying '*A explains B*'.

Definition 3.9. A *(regular) production inference relation* is a binary relation $\Rightarrow$ on the set of classical propositions satisfying the following postulates:

(Strengthening) If $A \vDash B$ and $B \Rightarrow C$, then $A \Rightarrow C$;
(Weakening) If $A \Rightarrow B$ and $B \vDash C$, then $A \Rightarrow C$;
(And) If $A \Rightarrow B$ and $A \Rightarrow C$, then $A \Rightarrow B \land C$;
(Cut) If $A \Rightarrow B$ and $A \land B \Rightarrow C$, then $A \Rightarrow C$;
(Truth) $\mathbf{t} \Rightarrow \mathbf{t}$;
(Falsity) $\mathbf{f} \Rightarrow \mathbf{f}$.

From a logical point of view, the most significant 'omission' of the above set of postulates is the absence of reflexivity $A \Rightarrow A$. It is this feature that creates a possibility of nonmonotonic reasoning.

Production rules are extended to rules with sets of propositions in premises as follows: for a set u of propositions,

$$u \Rightarrow A \;\equiv\; \bigwedge a \Rightarrow A, \text{ for some finite } a \subseteq u.$$

Let $\mathcal{C}(u)$ denote the set $\{A \mid u \Rightarrow A\}$. The operator $\mathcal{C}$ is monotonic and continuous, and it plays the same role as the usual derivability operator for consequence relations. However, this operator is not reflexive, which creates an important distinction among theories of a production relation.

[3]The origins of this result are in the Main Lemma from (Lifschitz & Schwarz 1993).

Definition 3.10. A *nonmonotonic semantics* of a production inference relation is the set of all its *exact theories*, namely sets u of propositions such that $u = \mathcal{C}(u)$.

An exact theory describes an informational state in which every proposition is *explained* by other propositions accepted in this state. Accordingly, restricting our universe of discourse to exact theories amounts to imposing a kind of an explanatory closure assumption on intended models.

It turns out that rules $A \Rightarrow B$ are representable as bisequents $\Vdash B : A$ of a biconsequence relation. More precisely, let us define the *production subrelation* $\Rightarrow_{\Vdash}$ of a biconsequence relation $\Vdash$ as the following set of production rules:

$$\{A \Rightarrow B \mid \Vdash B : A\}.$$

Then we have the following embedding:

Lemma 3.9. *$\Rightarrow$ is a production inference relation if and only if it is a production subrelation of some default biconsequence relation.*

The above correspondence can be extended to the correspondence between the associated nonmonotonic semantics. Namely, exact theories of a production inference relation coincide with the extensions of the associated biconsequence relation. Note, however, that biconsequence relations are not determined uniquely by their production subrelations, and hence constitute a more general logical formalism.

An especially interesting class of production inference relations is formed by causal inference relations defined as follows:

Definition 3.11. A production inference relation is *causal* if it satisfies

(Or) If $A \Rightarrow C$ and $B \Rightarrow C$, then $A \vee B \Rightarrow C$.

The rule Or sanctions reasoning by cases, and hence causal inference relations can be seen as systems of reasoning about complete worlds. Moreover, production rules of a causal inference relation can already be interpreted as truly causal rules, since they provide a natural representation of ordinary causal assertions. Also, the nonmonotonic semantics of such inference relations provide an exact formalization of reasoning in causal theories of action and change.

We will define now a class of biconsequence relations related to causal inference.

Definition 3.12. A default biconsequence relation will be called *causal* if it satisfies

(Negative Completeness) $: A, \neg A \Vdash .$

Negative Completeness restricts the assumption contexts to worlds, and hence a semantic representation of causal biconsequence relations can be given in terms of bimodels of the form (u, α), where α is a world. This implies, in particular, that any theory of a causal biconsequence relation should be a world.

Causal biconsequence relations satisfy all the rules and conditions for classical inference with respect to the assumption context of bisequents. In particular, we have the following reduction that eliminates negative conclusions in bisequents:

$$a : b \Vdash c : d \;\equiv\; a : \bigwedge d \to \bigvee b \Vdash c : .$$

It can be shown that causal biconsequence relations constitute an exact non-modal counterpart of Turner's logic of universal causation (UCL) from (Turner 1999).

The next result shows that any causal biconsequence relation generates a causal inference relation.

Lemma 3.10. *$\Rightarrow$ is a causal inference relation iff it is a production subrelation of some causal biconsequence relation.*

The above embedding demonstrates that the formalism of supraclassical biconsequence relations subsumes causal inference as a special case. This correspondence can also be extended to the associated nonmonotonic semantics. Note in this respect that the above elimination of negative conclusions implies that causal rules $A \Rightarrow B$ are representable also by bisequents $: \neg A \Vdash B :$, which correspond to ordinary prerequisite-free default rules of the form $: A/B$. The latter representation of causal theories in default logic can be found already in (McCain & Turner 1997b).

Finally, it can be shown that causal biconsequence relations constitute a maximal underlying logic for complete extensions. This is a syntactic counterpart of the semantic characterization of strong equivalence of UCL theories, given in (Turner 2004).

4 Default Logic Simplified

It has been shown above that generalized default logic subsumes many other nonmonotonic formalisms. Moreover, generalized default rules containing justifications in heads have played an important role in the representation of these formalisms. This makes still more surprising the reduction result we present in this section, which shows, in particular, that justifications in head of default rules can be eliminated. Furthermore, the result will show that default logic in its full generality is reducible to a quite simple and extremely natural formalism that involves only monotonic rules and assumptions.

To simplify the notation, the monotonic rules $a{:}/c{:}$ having no justifications neither in bodies nor in heads will be written below as ordinary inference rules a/c.

Definition 4.1. A generalized default theory will be called *simple* if it includes only rules of the following two kinds:

- Monotonic rules a/c;
- Supernormal defaults $:A/A$

We will describe now a translation of arbitrary default theories to simple ones. To begin with, we will extend the source propositional language $\mathcal{L}$ with new propositional atoms A°, for any classical proposition A in $\mathcal{L}$. For a set u of propositions from $\mathcal{L}$, u° will denote the set of new atoms $\{A^\circ \mid A \in u\}$.

Next, if D is a default theory in $\mathcal{L}$, then D° will denote the following set of rules in the extended language:

$$\{a, b^\circ/c, d^\circ \mid a : b/c : d \in D\} \tag{1}$$

plus the following rules for any formula A from $\mathcal{L}$ that appears as a justification in the rules from D:

$$\neg A/\neg A^\circ \qquad \text{and} \qquad : A^\circ/A^\circ \tag{2}$$

To begin with, it can be easily seen that the above translation is polynomial and modular. Moreover, the following theorem shows that this translation is also faithful, so it is actually a PFM translation in the sense of (Janhunen 1999).

Theorem 4.1. *A set u is an extension of D if and only if there is a unique extension u_0 of D° such that $u = u_0 \cap \mathcal{L}$.*

As many other essential results in nonmonotonic reasoning, the above theorem is also not completely new and has numerous precedents.

- From a technical side, the proof of the above result generalizes the proof of the corresponding result for disjunctive logic programs with negations in heads given in (Janhunen 2001).
- The first expression of this kind of reduction can be discerned from the representation of Reiter's default logic in terms of argument systems suggested in (Lin & Shoham 1989). In this representation, default rules $A{:}B_1, \ldots, B_n/C$ were represented as monotonic rules $A, \neg ab(B_1), \ldots, \neg ab(B_n)/C$. In addition, an argument system was required to contain monotonic rules of the form $\neg A/ab(A)$, plus *nonmonotonic* rules $\mathbf{t} \Rightarrow \neg ab(B)$, the latter rules having the same functionality as supernormal defaults $:\neg ab(B)/ab(B)$. As can be seen, for the case of Reiter's default logic our translation is just a notational variant of this representation.
- A further development of the argumentation approach to nonmonotonic reasoning has led to a general assumption-based framework for default reasoning suggested in (Bondarenko *et al.* 1997). The latter framework includes a deductive system of ordinary monotonic rules, a distinguished set of propositions called *assumptions*, and a mapping from assumptions to the set of all propositions of the language that determines the *contrary* of any assumption A. The authors have been able to demonstrate that default logic, as well as a number of other nonmonotonic formalisms, are expressible in this framework. In particular, the representation of default logic was based again on representing default rules $A{:}B_1, \ldots, B_n/C$ as monotonic rules $A, \mathbf{M}B_1, \ldots, \mathbf{M}B_n/C$, where propositions of the form $\mathbf{M}B$ were taken to be the assumptions, and $\neg B$ was considered a contrary to the assumption $\mathbf{M}B$.

  The above assumption-based representation is clearly similar to our translation to simple default theories. But in a sense, our reduction complements these representation results by showing that default logic is actually equivalent to such an assumption-based nonmonotonic framework.

5 Conclusions

It goes without saying that the ultimate aim of nonmonotonic formalisms consists in providing computational tools for solving the actual problems arising in Artificial Intelligence. Nevertheless, as in many other areas of scientific research, it should also be clear that our formalisms will have a chance to fulfil this aim only to the extent they will manage to provide an adequate, concise and versatile framework for *representing* such problems. In this sense, default logic can primarily be viewed as a framework for representing *defeasible knowledge*. The growing body of results on this subject indicates that default logic constitutes, ultimately, a proper unified framework for this task.

It is also well known that default logic is a computationally difficult formalism. It seems, however, that (as in any other honest business) this happens because we have to pay for the ability to handle and act on the basis of additional, defeasible knowledge that is not available in ordinary logical reasoning. This implies that the representation problems of nonmonotonic reasoning should to some reasonable extent be separated from the computability questions. In other words, we should know *what* ought to be expressed, or represented, even before we know *how* it could be computed. It seems to us that there should be no difference here between the theory of nonmonotonic reasoning and Logic in general.

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